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Y. Kozai

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Determination of Love's number from satellite observations

By Y. Kozai*

Smithsonian Astrophysical Observatory, Cambridge, Massachusetts, U.S.A. and Tokyo Astronomical Observatory, Mitaka, Tokyo, Japan

From variations of orbital inclinations of the three satellites 1959 α1, 1959 η, and 1960 ι2, Love's number of the Earth is determined as $0{\cdot}39 \pm 0{\cdot}05$.

The three satellites 1959 α1, 1959 η, and 1960 ι2 (see table 1) were adopted for this study simply because orbital elements determined from precisely reduced Baker-Nunn observations were available for long intervals of time for these satellites. Of the orbital elements, the inclination of the orbital plane to the equator is the most accurately determined, since, because of the geographical distribution of the S.A.O. astrophysical observing stations, many observations are made near apices.

Table 1. Satellites used in the determinations of Love's number

	a	i	e	$\dot{\Omega}$	
1959 α1 (Vanguard 2)	1·30	32·8°	0·16	−3·5°/day	Feb. 1959–May 1964
1959 η (Vanguard 3)	1·33	33·4	0·19	−3·3	Sept. 1959–Jan. 1964
1960 ι2 (rocket of Echo 1)	1·25	47·2	0·01	−3·1	Sept. 1960–June 1964

The orbital elements every 2 days are derived by using observations over 4-day intervals. Every possible perturbation is computed and subtracted from the orbital elements.

Besides lunisolar, solar radiation, and many other known perturbations, effects due to the motion of the Earth's equator should be taken into account. That is, since the orbital elements are referred to the moving equator, which is not a reference plane of the inertial system, perturbations are produced in some of the orbital elements (Kozai 1960).

The deformation of the Earth due to the Moon and the Sun causes perturbations in satellite orbits (Kaula 1962; Newton 1965; Kozai 1965). The most important perturbations for the inclinations of these three satellites are

$$0{\cdot}80^\circ \times 10^{-3} k_2 \cos(\Omega - \nu \Delta t), \quad \text{for 1959 } \alpha 1 \text{ and 1959 } \eta$$

and

$$0{\cdot}79^\circ \times 10^{-3} k_2 \cos(\Omega - \nu \Delta t), \quad \text{for 1960 } \iota 2,$$

where k_2 is Love's number, Ω is the longitude of the ascending node of the satellite, ν is the angular rotational velocity of the Earth, and Δt is the time lag of the tides due to the friction.

The perturbation terms with the argument Ω are produced by lunisolar gravitational attractions and the motion of the equator, and they should be computed carefully and subtracted from the inclination.

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From the residuals of the observed inclinations every 2 days the following periodic variations with argument Ω are derived:

$$0{\cdot}36^\circ \times 10^{-3} \cos(\Omega - 11^\circ) \quad \text{for 1959 } \alpha 1,$$
$$\pm 0{\cdot}07 \qquad\qquad \pm 10$$

$$0{\cdot}38^\circ \times 10^{-3} \cos(\Omega + 9^\circ) \quad \text{for 1959 } \eta,$$
$$\pm 0{\cdot}11 \qquad\qquad \pm 12$$

$$0{\cdot}28^\circ \times 10^{-3} \cos(\Omega + 23^\circ) \quad \text{for 1960 } \iota 2.$$
$$\pm 0{\cdot}06 \qquad\qquad \pm 11$$

By comparing the observed amplitudes with the theoretical values, values for k_2 are computed as

$$k_2 = 0{\cdot}45 \pm 0{\cdot}09 \quad \text{for 1959 } \alpha 1,$$
$$k_2 = 0{\cdot}47 \pm 0{\cdot}12 \quad \text{for 1959 } \eta,$$
$$k_2 = 0{\cdot}35 \pm 0{\cdot}06 \quad \text{for 1960 } \iota 2.$$

The mean value is $k_2 = 0{\cdot}39 \pm 0{\cdot}05.$

The time lag, Δt, for the tides can also be computed from the observed phase angle. However, since the scattering of Δt derived for the three satellites is large, Δt thus derived is not well determined, but is reasonably small.

The computation was made by HITAC 5020 at the Computer Centre, University of Tokyo.

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